CONTEMPORARY EDUCATIONAL PSYCHOLOGY **23,** 257–275 (1998)
ARTICLE NO. EP970962

Cognitive and Metacognitive Abilities Involved in the Solution of Mathematical Word Problems: Validation of a Comprehensive Model

Daniela Lucangeli, Patrizio E. Tressoldi, and Michela Cendron

Department Of General Psychology, Padova University, Padova, Italy

With this investigation we tried to validate an economic model of the cognitive and metacognitive abilities involved in Mathematical Word Problems. From the literature we chose seven abilities: text comprehension, problem representation, problem categorization, solution estimate, planning the solution, procedure self-evaluation, calculus self-evaluation. In order to measure these abilities, we devised a series of mathematical word problems (partitioned problem) where, for each ability, subjects have to answer a multiple choice question. A hierarchical regression model with only five variables (excluding solution estimate and calculus self-evaluation) explained more than 50% of the variance of the solution scores of the Partitioned Problems, and 40% of the variance of the solution of a series of problems presented with only the text. The model was validated with five different samples of subjects of different school grade and with a bootstrap procedure. Furthermore the relationship of the variables was tested using a path analysis. The discrimination validity of three different levels of problem solving efficiency give further support to the validity of the model. The model obtained may be utilized in order to devise practical instruments for the analysis of mathematical word problem difficulties.

INTRODUCTION

Which are the main cognitive and metacognitive abilities contributing to the solution of mathematical word problems? What is their influence on the solution?

In the present investigation we tried to validate a model of the abilities concurring in the solution of mathematical word problems, testing five different samples of subjects from the third to the seventh grade.

The literature on mathematical word problems (MWPs) is very rich, but there are few empirical investigations aimed at integrating the diverse cognitive and metacognitive abilities found to affect MWPs solution in a single comprehensive model.

Address correspondence and reprint requests to Patrizio E. Tressoldi, Dipartimento di Psicologia Generale, Università di Padova, via Venezia,8, 35131 Padova, Italy. Fax: + 49 + 8276600. E-mail: tressold@psico.unipd.it

0361-476X/98 $25.00

Riley and Greeno (1988); Riley, Greeno, and Heller (1983); and Polson and Jeffris (1982, 1985) suggest a distinction between two main components: a cognitive representation of the information drawn from the text and a definition of the procedures and the strategies necessary to attain the solution.

Mayer, Larkin, and Kadane (1984) consider four abilities:

- Transformation, of each sentence of the text into a mental representation;
- Integration, of the different information in a single coherent representation of the problem;
- Planning of the steps necessary to arrive at the solution;
- Execution of the plan for the solution.

Fasotti (1992) suggests the presence of at least two distinct components from his neuropsychological investigations with neurological patients. For example he found that, differently from patients with frontal lesions, patients with left-posterior lesions have great difficulty in the understanding of words expressing relations (i.e., sells 12 less than . . .), complex prepositions (i.e. sells 10 apples per day . . .). However in a problem categorization task, both types of patients based the sorting of the problems on the superficial text-characteristics (i.e. objects, actions) and not on the structure of solution.

Swanson, Cooney, and Brock (1993) found that solution ability in MWPs was correlated with measures of working memory, problem classification, knowledge of processing operations, reading comprehension and verbatim recall of word problem text. Among these variables those which contributed more to solution accuracy were reading comprehension and knowledge of processing operations. Problem classification and working memory were significant predictors only when forced in the equation first.

From the metacognitive point of view, Pressley (1990) and Montague (1992) underline the role of procedure of control such as, monitoring the comprehension, controlling the execution plan, evaluating the results.

From these contributions a certain agreement seems to emerge about which abilities contribute most to the solution of MWPs. We decided to choose the six (apart the solution phase) that received the strongest empirical support: text comprehension, problem representation, problem categorization, result estimation, and planning and self-evaluation that we divided into self-evaluation of the procedure and self-evaluation of the calculation, for a total of seven components.

The semantic comprehension of the text of the problem, the translation phase according to Mayer et al. (1984) terminology, requires most of the cognitive processes necessary for the comprehension of every other text, argumentative, narrative etc. plus some special knowledge about the meaning of some mathematical terms, such as ‘‘altogether,’’ ‘‘more than,’’ ‘‘less than,’’ etc.

For example Cummins (1991), demonstrated that the expression ‘‘Mary has 5 more marbles than John'' is interpreted as Mary has 5 marbles,'' suggesting that ''. . . children possess at least tacit understanding of part-whole relations, and what they come to learn through instruction or further familiarization with language is how certain verbal formats map onto those relations.'' Low, Over, Doolan, and Michell (1994) confirm the importance of text comprehension training their students to detect necessary and sufficient information from algebraic word problems.

Another ability that seems specific to mathematical problem solving is problem representation, that is, the construction of a mental model. According to Mayer (1981, 1992), information drawn from the text is connected (integrated) in a unified structure where the value of the different variables becomes related to each other and to the unknown data. This ability seems crucial or at least extremely important for guiding the future choices along the solution of the problem. It is intuitive that a wrong or a partial representation of the relation between the different variables and their value may heavily influence the solution plan and the calculation choices. There is a wide debate on which are the characteristics of this mental representation. Some authors argue that it is in a propositional format, others in pictorial format. Probably the format varies according to the type of problem and type of data and surely according to the expertise of the problem solvers (Nathan, Kintsch, & Young 1992). However, there is a certain consensus that visual representation plays an important role in the organization of the information given in the text and consequently on the comprehension of their relation favoring the ideation of the solution plan (Wicker, Weinstein, Yelich, & Brooks, 1978; Kaufman 1988; Lewis 1989; Antonietti, 1991; Hegarty, Mayer, & Monk, 1995).

A third ability, even if there is a debate whether it is always involved, is the capacity to categorize the problem, that is, the capacity to recognize its deep structure. For example, the problem is of the same type even if in a case it is Pete who gives 4 marbles to Mary and in another case it is Mom who gives 4 marbles to Jimmy. This ability has been found in the problem solver experts (Hinsley, Hays, & Simon, 1977; Larkin, McDermott, Simon, & Simon, 1980; Chi, Feltovich, & Glaser, 1981; Vanlehn, 1989) probably as a consequence of their experience with similar problems with school experience. In fact, Morales, Shute, and Pellegrino (1985) observed that fifth and sixth graders sort a series of problems according to a schema theory of problem representation and solution, whereas third graders sort the problems according to their surface description. Swanson et al. (1993) found that in third and fourth graders, the results in problem classification added a separate contribution to problem solution with respect to other variables such as reading comprehension and text recall. Rudnitsky, Etheredge, Freeman, and Gilbert (1995) demonstrated that this ability can be trained in order to facilitate the recovery of the inner structure of different typologies of addition and subtraction word problem by a method called structure-plus-writing. Subjects of the third and fourth grade, trained with this method, outperformed their controls not only at the end of the training but also at a follow up.

The solution plan is a fourth ability that is recognized as necessary in every word mathematical problem requiring at least two steps in order to arrive at the solution.

From the metacognitive analysis of the abilities involved in MWPs, we considered three of those that may be easily operationalized: estimation (approximately) of the result, self-evaluation of the procedure, and self-evaluation of the calculation. To estimate the solution may be considered a metacognitive ability to the extent that it involves a recall of past personal experiences with number calculations and with similar problems. Self-evaluation is the ability to monitor our own performance considering our own skills in solving MWPs in general and that particular kind of problem. Testing the efficacy of strategies education on MWPs, Montague et al. (Montague & Bos, 1986; Montague, 1992) demonstrated that learning-disabled children improved their performance after a training comprising strategies such as read to understand, paraphrase, visualize, hypothesize a plan to solve the problem, estimate, compute the calculations, and check their procedure and calculus.

The examination of the literature led us to formulate a comprehensive model of the variables involved in the solution of MWPs and to attempt its validation.

The model we chose to test includes the following components: text comprehension, problem representation, problem categorization, result estimation, planning the steps toward the solution, self-evaluation of procedure, and self-evaluation of calculations.

If we succeed in demonstrating that most of the variance of the solution may be explained by the different abilities we have chosen, we can use these data to prepare a clinical instrument for a finer analysis of the difficulties in the solution of MWPs that may offer important information for educational intervention.

METHOD

Subjects

We tested the following subjects: 64 subjects attending the 3rd grade; 78 subjects attending the 4th grade; 68 subjects attending the 5th grade; 67 subjects attending the 6th grade; 89 subjects attending the 7th grade. They were drawn from four different schools located in the north of Italy and most subjects were tested with the entire class. Their mathematical textbooks and teaching methods were quite similar, following the national elementary and middle school curricula. Mathematics lessons lasted 3 h a week.

All subjects were free from any mental handicap and sensory problems. The sociocultural background was prevalently middle class, with a mean of 8 years of schooling for their parents, similar to what it is expected from people of the same social class.

Materials

For each elementary class, 10 couples of word problems (only 8 couples for the 6th and the 7th grade) matched for their deep structure were chosen from their textbooks and accepted after an agreement with mathematics teachers.

The 10 Standard Problems were transcribed on a single sheet of paper. For each of the other 10 Partitioned Problems, four answers were prepared for each of the problem solving components chosen for the present investigation: text comprehension, problem representation, problem categorization, result estimation, solution plan, procedure self-evaluation, and calculation self-evaluation. Particular attention was paid not to give suggestions on the calculations necessary for the solution.

Example of a Mathematical Word Problem for the Fourth Grade

Standard version. Margareth told Paula she had 30 picture-cards of Lion King. Paula said she had 7 less than Rose. Rose on the contrary said she have 5 picture-cards more than Margareth. How many picture-cards have the three girls altogether?

Partitioned version. In an elementary school there are 3 fourth grade classes: 4th A, 4th B and 4th C. In the 4th C there are 20 children, in the 4th B there are 6 children less. In the 4th A there are 3 children more than in 4th B. How many children altogether ?

COMPREHENSION

Choose the sentence with the information more important for the solution:

☐ In the 4th B there are 6 children less than in the 4th C and 3 less than in the 4th A. (correct)

☐ In the school there are classes with different numbers of children. (irrelevant)

☐ In the 4th B there are 6 children more than in the 4th C. (wrong)

☐ In the 4th A there are 3 children less than in the 4th B. (partial)

REPRESENTATION

Choose among the pictures the one which correctly represents the problem (Fig. 1).

CLASSIFICATION

Which of the following problems could you solve in the same way as the problem you are working with?

☐ Mark is 6 years older than John. John is 2 years younger than Mary. How old is John? and Mary? (partial)

☐ In the 4th C and in the 4th B classes there are 32 children. How many teachers are there? (irrelevant)

☐ In one class there are 14 males and 6 females. If they use double seats, how many desks are there? (wrong)

☐ July has 19 picture-cards, her cousin has 6 more than her but Alice has 3 less than the cousin. How many picture-cards do the three friends have? (correct)

PREDICTION OF THE RESULTS

Choose which of the following possible sentences is getting nearer to the correct result:

☐ The children are less than 20. (wrong)

☐ The children in the 4th A and 4th B classes are about 40. (partial)

☐ Overall there are less than 60 children. (correct)

☐ We know only the number of the fourth grade children. (irrelevant)

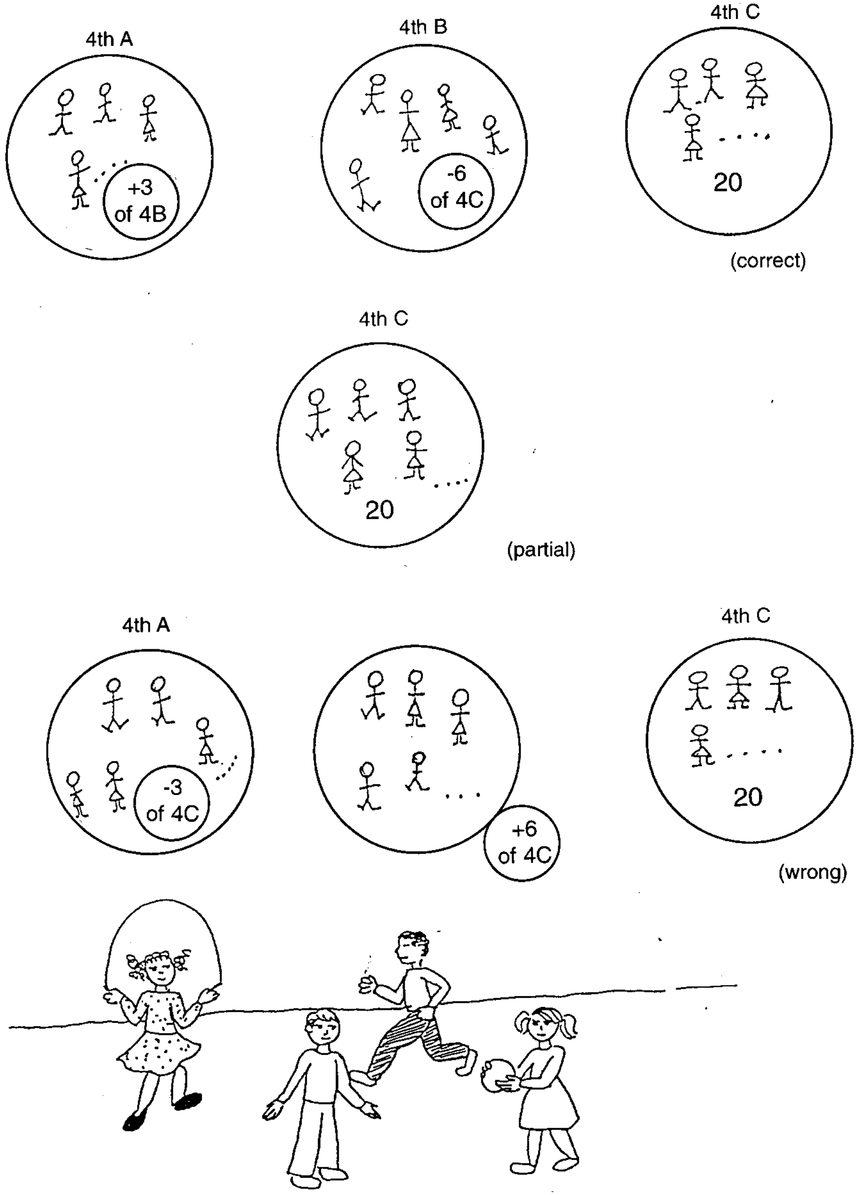


FIG. 1. The four choices of the representation component of the problem.

SOLUTION PLAN

Sign how you would solve the problem, ordering the sequences and numbering them from 1 to 3.

☐ I'll find out how many children there are in the 4th A .
☐ I'll find out how many children there are in the 4th B.
☐ I'll find out the sum of the children in the 4th A, 4th B and in 4th C.

EXECUTION

Solve the problem.

EVALUATION OF THE PROCEDURE

Sign how sure you are of having correctly executed your solution plan:

☐ I'm completely sure that I have correctly solved it.
☐ I'm unsure of having done it right.
☐ I'm unsure of having done it wrong.
☐ I'm sure that I have done it wrong.

EVALUATION OF THE CALCULUS

Sign how much you are sure of having correctly executed the calculus:

☐ I'm completely sure that I have correctly done it.
☐ I'm unsure of having done it right.
☐ I'm unsure of having done it wrong.
☐ I'm sure that I have done it wrong.

See another full example for the 6th class in the Appendix.

Scoring

For the first four components the four choices represent different degrees of accuracy:

- Correct answer (rated with four points);
- Partial answer (rated with three points);
- Wrong answer (rated with two points);
- Irrelevant answer (rated with one point);

We chose to rate less the irrelevant choice with respect to the wrong choice, because we consider it more severe to refer to information not related to the problem goal than to fail to comprehend its deep structure.

The solution plan was evaluated according to the number of steps ordered correctly (minimum zero, maximum according to the steps indicated in the problem).

The solution was also rated along a four level scale:

- Correct solution (four points);
- Correct procedure but calculation errors (three points);
- Partial solution (two points);
- Wrong solution (one point).

The two evaluation components were rated as follows:

- Three points in the case of an agreement between evaluation (sure of a correct solution or sure of a wrong solution) and solution (three-four points or one-two points, respectively);
- Two points in the case of the choice ''uncertain of having solved it right'' and ''uncertain of having solved it wrong'';
- One point in case of mismatch between evaluation and solution;
- Omissions were rated zero point.

It results evident that the score is higher as the subject is able to correctly monitor his/her performance.

For each grade, the battery of problems represented a range of MWPs commonly taught at that level. We were not interested in testing specific types of problems for example following Riley, Greeno, and Heller classification (1983), but in devising a battery to validate a model valid for the common problems faced by students at that grade. For example, in the third grade the problems required only to apply additions, subtractions and simple multiplication and some knowledge on the relationship between meters and kilometers. In the seventh grade, some problems required geometric knowledge such as how to solve the perimeter or the area of triangles and the capacity to use all the arithmetic calculations, fractions, proportions.

All problems required at least three steps and two different arithmetic calculations for the solution.

Procedure

During the second term of their grade, the problems were presented to the subjects during the normal school activity by their teachers with the presence of a research assistant. One day the Standard Problems were presented for solution and the subjects were asked to read the text carefully and solve them according to their preferred mode. Another day, the Partitioned Problems were presented, and the subjects were asked to answer the four choice options of the first four problem-solving components, solve the problem, and complete the two evaluation questions.

RESULTS

Reliability

For each class the Cronbach's α and the Guttman Split-half coefficient were calculated in order to verify the reliability of the problem battery. The Cronbach's α ranged from .80 to .90 with a mean of .84, the Guttman split-half, ranged from .69 to .83 with a mean of .74. In the Appendix we give the details for each class.

The values of both coefficients are rather satisfactory, confirming the goodness of the internal reliability of each battery of problems.

Concurrent Validity

To have a measure of the concurrent validity, we compared the mean obtained at the solution of the two types of problems, those solved after a series of response choices, and those solved using traditional procedures.

In Table 1 we report the means of the scores obtained in the solution of all the problems (maximum score for each problem = 4), their standard deviation, the correlation and effect size of the two types of problems for each class.

Discussion. All the correlation coefficients are very high and statistically significant, confirming that the two versions of the problems share a significant percentage of variance.

The comparison of the mean scores obtained in the solution of the two

TABLE 1
Mean, Standard Deviation, Correlation, and Effect Size (d) of the Solution Score for the Two Types of Problems for Each Grade

Grade	Standard problems Mean	(*SD*)	Partitioned problems Mean	(*SD*)	r	Effect size
3rd	2.33	(.81)	2.14	(.70)$^+$	.80*	.24
4th	2.96	(.70)	2.91	(.78)	.75*	.06
5th	2.08	(.88)	1.93	(.95)$^+$	.81*	.16
6th	2.13	(.76)	2.32	(.82)$^+$	.65*	−.23
7th	2.45	(.94)	2.28	(.95)$^+$	.55*	.18
Grand mean	2.42	(.86)	2.33	(.9)$^+$	.73*	.10

* $r = p < .05$; $^+t = p < .05$.

versions, reveals that the mean obtained in the Partitioned Problems does not always result lower than those obtained with the Standard Problems and that the effect sizes of the differences are constantly low. This result suggests that the Partitioned Problems do not penalize or do not facilitate the solution of the problems. We can then assume that during the solution of the Partitioned Problems, subjects utilize the same cognitive mechanisms employed in the solution of the Standard Problems.

Common Variance between the Solution and the Cognitive Components

The principal aim of the investigation concerns the investigation of the shared variance between the solution and the seven cognitive components, searching for an economic model, that is a model with the maximum shared variance and the minimum number of variables.

Simply for descriptive purposes we report the intercorrelations among all the variables in the Appendix.

We then analyzed the data using a hierarchical multiple regression, taking the scores of the seven cognitive variables as independent variables and the score of the solution as dependent variable in the order as follows: text comprehension, problem representation, problem categorization, result estimate, solution plan, procedure self-evaluation, and calculation self-evaluation.

The mean and standard deviation (in parentheses) of R^2 increment obtained from the six samples of subjects, for each variable, was: Comprehension, .28 (.17); Representation, .10 (.09); Categorization, .08 (.04); Estimate, .02 (.02); Solution Plan, .09 (.07); Procedure Self-Evaluation, .06 (.02); Calculation Self-Evaluation, .002 (.001). From these results we found that result estimation and calculation self-evaluation did not add a significant increment of variance. For the subsequent validations we then used a model with only

TABLE 2
Mean and Standard Deviation of Multiple Correlation R^2 after 100 Bootstrap Resampling for Each Grade

	Partitioned problems		Standard problems	
Grade	Mean	*SD*	Mean	*SD*
3rd	.68	.06	.49	.09
4th	.81	.03	.55	.10
5th	.83	.05	.55	.08
6th	.68	.06	.43	.10
7th	.50	.08	.30	.07

five cognitive components, text comprehension, problem representation, problem categorization, solution plan, and procedure self-evaluation.

To control the stability of this reduced model, for each grade we calculated the mean and the standard deviation of R^2 obtained with the hierarchical multiple regression analysis after 100 resampling using a bootstrap procedure (Péladeau, Lacouture 1993). These results are presented in Table 2.

Discussion. For the Partitioned Problems the five cognitive components, text comprehension, problem representation, problem categorization, planning the solution and self-evaluation of the procedure, explain more than the 50% of the variance of the solution. This result confirms that these cognitive abilities are involved in the solution phase of the problem and all together give an important contribution to its quality.

Even if to a lesser degree, this relationship is valid also for the Standard problems, giving a further support to the validity of our partitioned model. The minor degree of shared variance with respect to the Partitioned Problems may be easily explained by the fact that during the solution of Standard Problems, some subjects may bypass some of the cognitive components if they are not forced to use them.

Path Analysis

In order to test the mutual relationship among the variables of the model, we tested the goodness of fit of different relationships maintaining the comprehension as the first variable and the solution as the last, using the EQS program (Bentler 1995). The model which gives the best fit indices is the model presented in Fig. 2, where comprehension influences directly the representation, the categorization and the planning of the solution without postulating a mutual relationship among these variables.

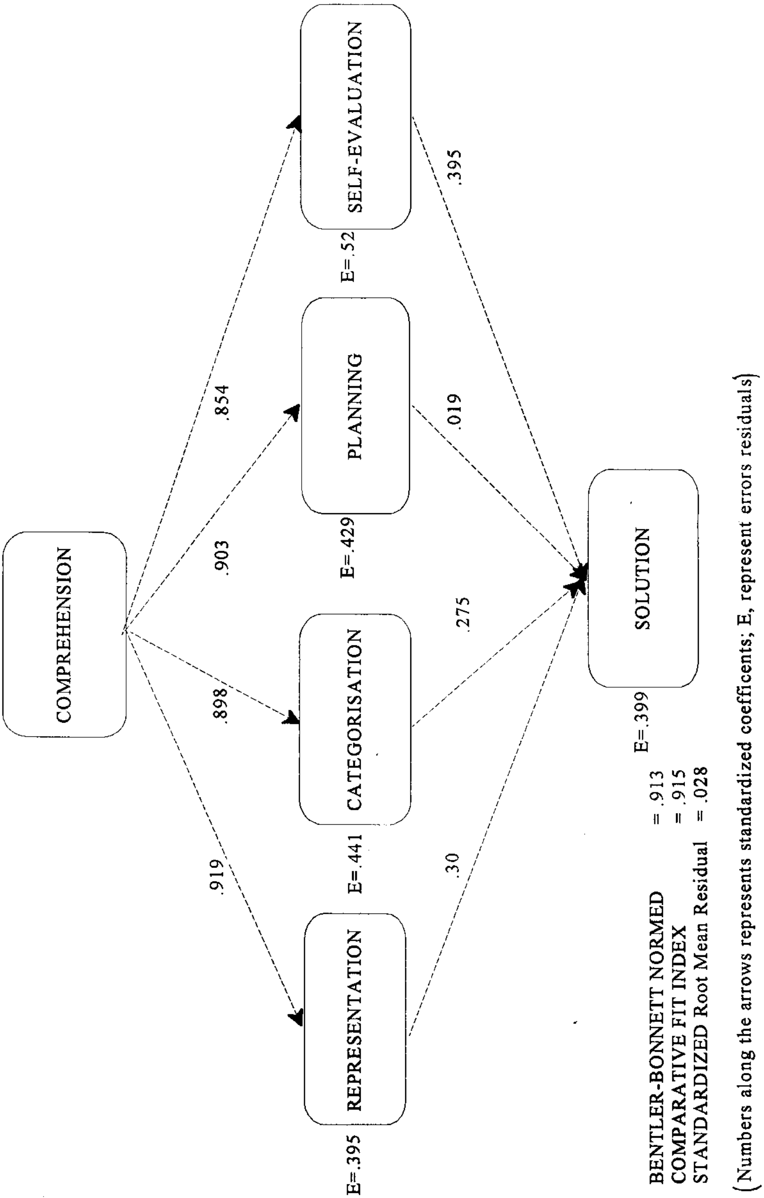


FIG. 2. Results of the path analysis.

TABLE 3
Prediction Percentages of the Three Problem-Solver Categories Obtained with a Discriminant Analysis Using the Jackknife Method

Ability category	Bad solvers prediction	Medium solvers prediction	Good solvers prediction
Bad solvers	**77.2**	32.8	
Medium solvers	9.5	**76.5**	14
Good solvers		27.2	**72.8**

Note. Results are means of the five grades.

Discussion. The Fit indices appears quite good, suggesting that after the comprehension phase every other component adds a unique contribution to the solution. The sum of this contribution is crucial for the result of the solution.

Discriminant Validity

To control the discriminant validity of the variables retained in the regression model, we tested the degree of discrimination of different levels of solution efficiency. We divided the subjects into three categories according to their solution score in the Partitioned Problems. The three categories corresponded respectively to the first quartile (bad solvers), to the two interquartiles (medium solvers) and the fourth quartile (good solvers).

In the Table 3 we report the rounded mean of percentage of categories prediction for all grades, obtained with a discriminant analysis using the jackknife[1] method (Huberty 1994), using the five cognitive components retained in the model as dependent variables. In the Appendix we report the same analysis divided for the five grades.

Discussion. The percentages of the hits are not particularly impressive even if they are always substantially superior to the chance of 33%, using the control for the maximum chance criterion (Huberty 1984). However the discriminant validity of the five variables receives support from the fact that the cases not predicted in the expected category fall only in the adjacent categories. Obviously more refinements are needed in the choice of the problems within the battery to improve the discrimination among the three categories of problem solvers.

GENERAL DISCUSSION

The aim to validate an economic model of the abilities contributing to the solution of MWPs, seems to have been reached. Using a procedure of MWPs

[1] The jackknife method involves conducting separate analysis dropping one different subjects of the sample from each one. The final statistics are the means of the different results obtained in the *n*-1 analysis.

solution requiring to answer different questions related to the text comprehension, problem representation, problem categorization, then to order the steps of the solution plan, and, after the calculations for the solution, to evaluate the procedure utilized during this last phase, we obtained a model explaining more than 50% of the solution variance. Further variance is surely due to the calculation abilities and to specific mathematical knowledge for example how to calculate the area of a triangle, or how to calculate a percentage. According to Swanson et al. (1993) other variance is surely due to not specific variables like recall of text information and working memory capacity.

The reliability of this model is substantiated by the replication with five different samples of different grades and the use of the bootstrap method.

Its validity is supported by the similar results obtained with Standard Problems and by the results obtained with the discriminant analysis.

The choice of a hierarchical model in the regression analysis is not sufficient to know how text comprehension affects problem representation, problem categorization and the other variables. The path analysis gives an answer to this question suggesting that after the comprehension of the relevant information embedded in the text, the capacity to use a representation, to categorize the problem, to plan and to have a good capacity of self evaluation may separately contribute positively to the solution.

From our data, it is then reasonable to affirm that the solution of a MWP requires a constellation of different cognitive and metacognitive abilities. Five of these seem very important: the semantic comprehension of the relevant information in the text, the capacity to have a good visual representation of the data, the capacity to recognize the deep structure of the problem, the capacity to order correctly the steps to arrive at the solution and a good capacity to evaluate the procedure utilized in the solution.

The first two abilities have received many confirmations from the literature, partly cited in the introduction. The ability to categorize the problems according to their deep structure has been less investigated in children even if its importance seems evident. However it has not been confirmed if this ability may give a special contribution to the solution of the problem or if it is an optional. Our data and the results obtained by Rudnitsky, Etheredge, Freeman and Gilbert (1995) suggest that it is very important.

The role of self-evaluation is quite particular. As a metacognitive ability it requires a competence to correctly evaluate the goodness of the procedures utilized in the solution phase. The more the subject is correct in the relationship between the result obtained and its evaluation, the more he/she demonstrates a good level of metacognitive knowledge on the choices adopted in the solution. It is plausible to imagine that in the case of wrong solutions, this knowledge is the basis for searching how to recover what it is lacking and to search how to improve.

The fact that the self-evaluation of the calculation did not add further variance, may be easily explained by the high correlation with the self-evaluation of the procedure (.90), probably because subjects include the efficacy of calculation in the evaluation of the procedure.

The educational implications of our result are very interesting. First of all we have sufficient information to derive a battery for the evaluation of the abilities involved in MWPs of subjects with difficulty in this academic area. After this evaluation it is possible to obtain more precise information on the efficiency of the five abilities tested with the battery in order to suggest more precise techniques to train those resulting not adequate.

There are already some suggestions on how to train students to improve their problem-solving efficiency. For example, Lewis (1989) and Willis and Fuson (1988) improved the problem-solving performance of their subjects by teaching them how to represent arithmetic word problems using diagrams or schematic drawings. Instead, Low et al. (1994) trained students to identify necessary and sufficient information, whereas Lopez and Sullivan (1991) studied the effects of personalizing the text incorporating concrete and familiar situations. Perhaps the most comprehensive curriculum is that tested by Montague (1992) with learning-disabled middle school students. The curriculum comprehends all the five cognitive abilities tested in our model plus others: comprehending the text, paraphrasing the information, visualizing the problem, planning the steps to the solution, estimating the results, computing the calculation, and evaluating the results. The curriculum is completed with a series of metacognitive strategies like self-instruct, self-question, and self-monitor.

While our MWPs comprehensive model obviously requires further refinements, we think that its present formulation might be of some interest to all those who are interested in the analysis of MWPs difficulties.

APPENDIX

Cronbach's Alpha and Guttman Split-Half Coefficients for Each Battery of Problems

Grade	Cronbach's α	Guttman Split-half
3rd grade	.85	.77
4th grade	.83	.69
5th grade	.90	.83
6th grade	.80	.69
7th grade	.82	.74
Mean	**.84**	**.74**

Prediction Percentages of the Three Problem Solver Categories Obtained with a Discriminant Analysis Using the Jackknife Method for Each Grade

Ability category	Bad solvers prediction	Medium solvers prediction	Good solvers prediction
		3rd Grade	
Bad solvers	**59**	41	
Medium solvers	9	**82**	9
Good solvers		34	**66**
		4th Grade	
Bad solvers	**82**	18	
Medium solvers	10	**81**	9
Good solvers		24	**71**
		5th Grade	
Bad solvers	**89**	11	
Medium solvers	9	**81**	10
Good solvers		28	**72**
		6th Grade	
Bad solvers	**77**	23	
Medium solvers	10	**75**	15
Good solvers		31	**69**
		7th Grade	
Bad solvers	**79**	21	
Medium solvers	10	**64**	26
Good solvers		14	**86**

Intercorrelations among the Different Components

	COMPR	REPR	CLASS	PREDIC	PLAN	SOLUT	PROC-EVAL
REPR	.7659						
CLASS	.6855	.6720					
PREDIC	.7047	.6610	.6086				
PLAN	.5894	.6693	.6562	.5381			
SOLUT	.5767	.6453	.6598	.5886	.6991		
PROC-EVAL	.3382	.3719	.3193	.3277	.3058	.3243	
CALC-EVAL	.3061	.3553	.2644	.3180	.2792	.3051	.8968

Note. COMPR, Comprehension; REPR, representation; CLASS, classification; PREDIC, prediction; PLAN, solution plan; PROC-EVAL, procedure evaluation; CALC-EVAL, calculation evaluation.

Example of a Mathematical Word Problem for the Seventh Grade

Standard version.
The sum of the two different sides of a rectangle is 46,1 dm. One of the two sides is 45 cm longer than the other. Find the area of half rectangle.

Partitioned version.
The sum of the base with the height of a rectangle, is 17,2 meters and their difference is 3,6 decimeters. Find the area of the triangle.

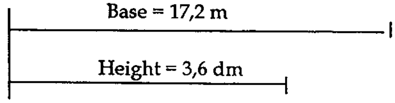
COMPREHENSION

Choose the sentence with the information more important for the solution:
☐ In a triangle, the base and its height differ by 3,6 dec and together sum to 17.2 meters. (correct)
☐ The base of a triangle is 17,2 meters and its height, 3,6 dec. (wrong)
☐ In a triangle there are always three bases and three heights. (irrelevant)
☐ In a triangle the base and its height sum to 17,2 meters. (partial).

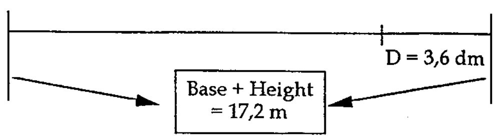
REPRESENTATION

Choose among the diagrams which one correctly represents the problem.

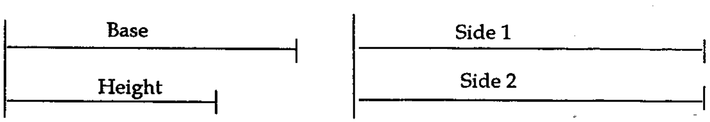
☐ wrong



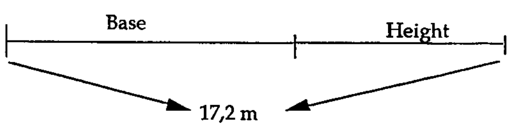
☐ correct



☐ irrelevant



☐ partial



CLASSIFICATION

Which of the following problems could you solve in the same way as the problem you are working with?
☐ In a scalene triangle the perimeter is 28 cm. and the height 11 cm. Find the area of the triangle. (irrelevant)

☐ Find the area of a triangle whose base is 21 meters and the height 15 meters. (partial)
☐ In a rectangle the base plus the height sum to 18 cm., and their difference 4 cm. Find the area of half rectangle. (correct)
☐ Find the area of a equilateral triangle knowing that the perimeter is 24 cm., and the height 15 cm. (wrong)

SOLUTION PLAN

Sign how you would solve the problem, ordering the sequences and numbering them from 1 to 4.
☐ I find out the measure of the height;
☐ I find out the area of the triangle;
☐ I find out the measure of the base;
☐ I find out the sum of the base and the height in decimeters;

EXECUTION

Solve the problem.

EVALUATION OF THE PROCEDURE

Sign how sure you are of having correctly executed your solution plan:
☐ I'm completely sure that I have correctly solved it.
☐ I'm unsure of having done it right.
☐ I'm unsure of having done it wrong.
☐ I'm sure that I have done it wrong.

EVALUATION OF THE CALCULUS

Sign how sure you are of having correctly executed the calculus:
☐ I'm completely sure that I have correctly done.
☐ I'm unsure of having done right.
☐ I'm unsure of having done wrong.
☐ I'm sure that I have done it wrong.

REFERENCES

Antonietti, A. (1991). Why does mental visualization facilitate problem solving ? In R. H. Logie & M. Denis (Eds.), *Mental images in human cognition.* Elsevier: Amsterdam.

Bentler, P. M. (1995). *EQS structural equations program manual.* Encino, CA: Multivariate Software Inc.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by expert and novices. *Cognitive Science*, **5,** 121–152.

Cummins, D. D. (1991). Children's interpretations of arithmetic word problem. *Cognition and Instruction,* **8**(3), 261–289.

Fasotti, L. (1992). *Arithmetical word problem solving after frontal lobe damage. A cognitive neuropsychological approach.* Swets & Zeitlinger, Amsterdam.

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology,* **87**(1), 18–32.

Hinsley, D., Hayes, J. R., & Simon H. (1977). From words to equation: meaning and representation in algebra word problems. In P.A. Carpenter, M. A. Just (Eds.), *Cognitive processes in comprehension.* Hillsdale: Erlbaum.

Huberty, C. J. (1984). Issues in the use and development of discriminant analysis. *Psychological Bulletin,* **95,** 156–171.

Huberty, C. J. (1994). *Applied discriminant analysis.* New York: Wiley.

Kaufman, G. (1988). Mental imagery and problem solving. In M. Denis, J. Engelkamp, J. T. E. Richardson (Eds.), *Cognitive and neuropsychological approaches to mental imagery* (pp. 231–240). Boston: Nijkoff.

Larkin, J. H., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Expert and Novice performance in solving physic problems. *Science,* **208,** 1335–1342.

Lewis, A. B. (1989). Training students to represent arithmetic word problems. *Journal of Educational Psychology,* **81,** 521–531.

Lewis, A. B., & Mayer, R. E. (1987). Students miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology,* **79,** 363–371.

Lopez, C. I., & Sullivan, H. J. (1991). Effects of personalized math instruction for Hispanic students. *Contemporary Educational Psychology,* **16,** 95–100.

Low, R., Over, R., Doolan, L., & Michell, S. (1994). Solution of algebraic word problems following training in identifying necessary and sufficient information within problems. *American Journal of Psychology,* **107**(3), 423–439.

Mayer, R. E. (1981). Frequency norms and structural analysis of algebra story problems. *Journal of Educational Psychology,* **74,** 199–216.

Mayer, R. E. (1992). *Thinking, problem solving, cognition.* New York: Freeman.

Mayer, R. E., Larkin, J. H., & Kadane, J. (1984). A Cognitive analysis of mathematical problem solving ability. In R. Sternberg (Ed.), *Advances in the psychology of human intelligence.* Hillsdale: Erlbaum.

Montague, M., & Bos, C. S. (1986). The effect of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. *Journal of Learning Disabilities,* **19**(1), 26–33.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities,* **25**(4), 230–248.

Morales, R. V., Shute, V. J., & Pellegrino, J. W. (1985). Development differences in understanding and solving simple mathematics word problems. *Cognition and Instruction,* **2**(1), 41–57.

Nathan, M. J., Kintsch, W., & Young, E. (1992). A theory of algebra word problem comprehension and its implications for the design of learning environments. *Cognition and Instruction,* **4,** 329–390.

Péladeau, N., & Lacouture Y. (1993). SIMSTAT: bootstrap computer simulation and statistical program for IBM personal computer. *Behavior Research Method, Instruments, & Computers,* **25**(3), 410–413.

Polson, P. G., & Jeffries, R. (1982). Problems solving as search and understanding. In R. J. Sterneberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 1). Hillsdale: Erlbaum.

Polson, P. G., & Jeffries, R. (1985). Instruction in general problem solving skills: An analysis of four approaches. In J. W. Segal, S. F. Chipman, & R. Glaser (Eds.), *Thinking and learning skills* (Vol. 1). Hillsdale: Erlbaum.

Pressley, M. (1990). *Cognitive strategy instruction that really improves children's academic performance.* Cambridge, MA: Brookline Books.

Riley, M. S., Greeno, J. G., & Heller, J. J. (1983). Developments of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking.* New York: Academic Press.

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction,* **5,** 49–101.

Rudnitsky, A., Etheredge, S., Freeman, S. J., & Gilbert, T. (1995). Learning to solve addition and subtraction word problems through a structure-plus-writing approach. *Journal of Research in Mathematics Education,* **26**(5), 467–486.

Swanson, H. L., Cooney, J. B., & Brock, S. (1993). The influence of working memory and classification ability on children's word problem solution. *Journal of Experimental Child Psychology,* **3,** 374–395.

Vanlehn, K. (1989). Problem solving and cognitive skill acquisition. In M. Posner (Ed.), *Foundations of cognitive science.* Cambridge, MA: MIT Press.

Wicher, F. W., Weinstein, C. E., Yelich, C. A., & Brooks, J. D. (1978). Problem reformulation training and visualization training with insight problems. *Journal of Educational Psychology,* **70,** 372–377.

Willis, G. B., & Fuson, K. C. (1988). Teaching children to use schematic drawings to solve addition and subtraction word problems. *Journal of Educational Psychology,* **80,** 192–201.