A Very Simple Quantum Model of Mind and Matter

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ABSTRACT

Since the Pauli-Jung conjecture of an ontological unity of Mind and Matter was formulated almost one century ago, there has been a lot of interest in trying to define a model based on the principles of Quantum Mechanics. In particular, Quantum Mechanics seems to offer a possibility to bridge the epistemological gap between Mind and Matter to build a model of what the 17th century philosopher Gerhard Dorn has called the "*Unus Mundus*", i.e. the ontological healing of the perceived duality of the Universe. Following other authors who have treated this matter before us, in the current paper we define a very simple model of Mind and Matter represented by two qubits and we discuss their interaction and their entanglement. In particular, we show how a generic Hamiltonian can be divided in four components. Two of them represent local evolutions of the qubits, each one acting on one of the two qubits and not on the other one. The other two define two different modes of interaction that we describe as the general evolution of the *Unus Mundus* and the interaction generating synchronicity. We conclude with a discussion on the different times governing the evolution of the Mind-Matter ensemble.

Key Words: Unus mundus, mind and matter, quantum information theory, entanglement, synchronicity

DOI Number: 10.14704/nq.2017.15.2.1031 **NeuroQuantology 2017; 2:186-199**

Introduction

Looking at the history of human knowledge, it is widely recognized that a definite turning point is the birth of modern science, which is crystallized by the works of Galileo.

His two fundamental pillars were science intended as "reasoned experience" and the assumption that "[the book of the universe] is written in mathematical language"[2]. While the *significance* of this revolution is universally acknowledged, as far as its *meaning* goes, we probably could use the remark of Chairman Mao when asked a similar question about the French Revolution: "it may be too early to tell".

While usually science historians concentrate on the undeniable progress that these two concepts have introduced, I would like to discuss what has been the *price* of this advance. By assigning a *language* and a *method* for the exploration of the Universe, Galileo has, perhaps unintendedly, also sparked an epistemic revolution that has pushed physics and metaphysics further, and perhaps irremediably, apart. A strong ontic interpretation of Galileo's statement is the injective relation between mathematics and reality.

Whatever is real can be expressed in mathematical terms and is accessible to "reasoned experience". Galileo does not say this, but we may

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Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

[2] `Philosophy is written in this grand book which stands continually open in front of our eyes (I mean the universe), but cannot be understood unless one first learns to comprehend the language and know the characters in which it is written. It is written in mathematical language, and the characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a word; without these one is wandering in a dark labyrinth." (Galileo Galilei, Il Saggiatore, Cap. VI)

speculate that the opposite may not be necessarily true, i.e. it is conceivable that with mathematics we can express concepts that are not true, as it is the case with language. The direct consequence of Galileo's predicate, however, is that all what cannot be expressed in mathematical terms does not belong to the *book of the universe*, i.e. it does not exist. One immediate problem with this is that what we intend with mathematics is also in continual evolution, and therefore the definition of reality seems to depend on the extent of the mathematical concepts acquired at the moment we consider it. In this perspective, it is interesting to note that we speak of mathematical *discoveries* and not *inventions*, as if mathematics, much like the Universe itself, had an ontic essence, i.e. it were *already* there for us to discover rather than invent.

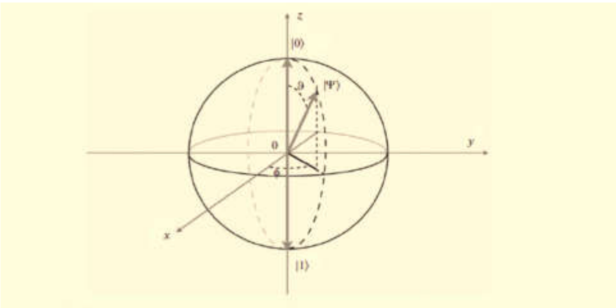

Figure 1: The Bloch sphere.

But we would like to advance the further hypothesis that what Galileo also suggested with his definition, was a renouncement of classical metaphysics, or rather the proposition of an imminent, if not materialistic metaphysics. As long as we speak the language of the Universe and we can describe it, we may as well stop bothering about its deeper *metaphysical* nature. There is no need to find a *transcendent explanation*: all is in the language and in the empiricist epistemology of the *reasoned experience*. Galileo here seems to foreshadow Wittgenstein or Lacan in identifying the language and the object spoken of. Religion and god are not negated explicitly; they are simply not needed any longer to understand the Universe. The implications of this on the religious and philosophical level were such that the Church could not tolerate them, and it used the relatively less important, but much more understandable by the public, question of the central position of Sun and Earth to silence him.

A *weaker* interpretation of Galileo's predicates could be that the only access we have to reality is via the empirical experience ordered by the mathematical language, while we are forever denied the knowledge of the ontic reality underlying. While perhaps more satisfying from the intellectual point of view, this second interpretation does not improve our position since it is little consolation to postulate something while at the same time say that we have no access to it.

The successes of physics and positive science are there under our eyes, technological revolution after revolution, such that there is no doubt of the effectiveness of Galileo's intuition. However, we are left in heritage also with an implicit assumption of a strong dependency, if not identity, between the model we use to describe reality and reality itself. The immense success of classical physics up until the end of the 19th century has somewhat pushed this question to the backstage. The fact that the nature could be described adequately by assuming absolute space and time, time-reversal, locality of interactions, Galilean invariance, commutativity, linear approximations, continuity and independence of the observed from the observed somehow lead us to believe that these were not only features of our model, but they were connected with fundamental characteristics of Nature itself. This explain why relativity, quantum physics and chaos have been perceived not only as empirical observations requiring us to modify our model, but as fundamental *epistemic revolutions* modifying our view of nature and our place within.

In *hindsight*, as much of a hindsight one may have on such matters at this moment, we could say that, blinded by the brilliance of the classical physics edifice we have considered that we were, at least intellectually, *maîtres et possesseurs de la nature*. Just one famous anecdote by Brush (1969) who reports that A. Michelson told him that: *At the end of the XIX century the professor of Physics of Max Planck suggested him to take piano lessons because there was nothing else to do in physics other than measure the constants of nature with a few more decimal places.* Brush (1969). The successive discoveries of the non-existence of ether, quantum effects, chaos, dark matter and dark energy, just to mention few, have turned what seemed a vast empire into a citadel surrounded by darkness on all sides. The forays into darkness have been realised at a high epistemological cost, having to forsake several concepts long held as “intuitive” and “natural”.

A more attentive analysis of the history of physics however shows us that the seeds of this revolution were sown long ago, and indeed since the beginning of the Galilean revolution. To realise this, let's hear what Einstein (1979) has to say about the discovery of restricted relativity: *By and by I despaired [verzweifelte ich] of discovering the true laws by means of constructive efforts based on known facts. The longer and the more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results*. This remark may look a complete and definitive departure from Galilean *reasoned experience*, since empiricism is forsaken in favour of a "formal cause" or "formal principle" directing Nature that cannot be deduced from facts, but that, once discovered, could illuminate the empirical world with a new meaning. But looking back to all the previous "grand syntheses", we see other formal principles being adopted with no or little empirical evidence. Galileo postulated the equivalence principle, i.e. the proportionality between inertial and gravitational mass, with very little evidence to support its universality. Newton postulated absolute space and time again as a formal principle, with little evidence to support this.

It is also interesting to realize how Newton's recursion method would lead centuries later to the development of the Julia's set, one of the first fractal sets discovered. Even more interesting, three Newtonian bodies exhibit an unstable behaviour that can only be completely accounted for in the framework of chaos theory. Many of the ideas that led to the theory of chaos were indeed expressed in the famous paper by Poincaré (1890), where he first described a chaotic deterministic system. Even in the Olympus of Newtonian mechanics, Dionysus was hiding.

This evolution of science has an almost Platonic element in it, insofar as progress is made when a "formal principle" akin to a Platonic *idea* is *discovered* (Plato would say *remembered*) to give meaning to the empirical experience. Popper has somehow sanctioned this process by giving to experience the role of *vetting* or *falsifying* theory rather than being its only source, leaving undetermined the path to arrive at a theory.

When quantum mechanics led us to abandon established and seemingly universal principles such as commutativity and the independence of the observed from the observer, this has been perceived as a profound cultural shock, epitomized by the *withdrawal* of Einstein, who has undeniably been one of the founding father of the quantum theory, with his famous sentence "God does not play dice". But if these reactions are understandable from the point of view of classical (and relativistic) physics, they seem to be hardly justified if we consider that physics as it stands today falls very short from explaining the living world, even if we have all confidence that it eventually will, and even farther from providing an explanation of what we can globally indicate as *Psyche* (from the Greek word ψυχή indicating the soul). The *historical* reaction to this empirical fact has been to argument that *because physics (as we understand it today) is based on principles incompatible with those of the Psyche* then *physical and psychical worlds must be different*. The flaw in this is, again, in the identification of the model with the modelled. The physical world *is not* the model we use to describe it, no matter how well this works. If the model cannot be extended to other elements of our empirical experience, the Psyche being one of them, we should rather consider the model, and not contradict Occam's principle (*Pluralitas non est ponenda sine necessitate*) by supposing a physical and psychical world separated one from the other.

In this context, quantum mechanics has been a real epistemic *game changer*. *Reasoned experience* has forced us to renounce some of the most cherished universal principles such as the Abelian nature of the world and the independence of the observed from the observer to explain the empirical evidence coming from the very basic building blocks of our physical world. It has also bestowed on us quantum entanglement and the EPR *paradox*. This has introduced at the most fundamental level of physics concepts that are commonplace in the psychological world and in the live sciences. The reductionist has now to admit that the whole physical world is *fundamentally* a quantum system, even if the effects may be washed away in the macroscopic world. This may well be *counter-intuitive*, but it removes part of the hurdles to consider a fundamental, ontic, unity between Physis (from the Greek word φύσις indicating the natural world) and Psyche.

This important development was very clear to the pioneers of Quantum Mechanic and psychoanalysis, who, after centuries of ontic dualism, finally saw the hope of a really *grand unification* between Physis and Psyche. The iconic exchanges between Jung and Pauli have set the stage for this relatively new discipline that we now call psychophysics. Jung has traced its origin back to the work of the alchemists and to the underground current of Gnosticism. In the light of what we said before, the tantalizing similarities between quantum physics and Psyche are no accident, but they are just an expression of their common ontic essence. What has to be considered interesting is rather the fact that a large portion of the physical world, the Galileo-Newton-Einstein universe, can be described with astounding precision using somewhat simplifying assumptions.

The concept of ontic unity of the perceptible world is often expressed in the terms used by the alchemist Gerhard Dorn in his work of 1602, who postulated the existence of a holistic reality he called *Unus Mundus*. Encompassing all reality, including what we used to call Physis and Psyche, this *Unus Mundus* is very likely non-Abelian, non-local and quantic, as the Psyche and the subject matter of quantum mechanics are. We also know that a portion of it can be described with good precision by Abelian, local and non-quantic models, such as classical and relativistic physics. These can be considered excellent approximations, in the same way in which Newtonian mechanics is perfectly adequate to describe the trajectory of a gun shell, without resorting to relativistic mechanics. We can suppose that this ontic entity is also the place where formal principles outside ordinary space-time operate, such as the Pauli exclusion principle and the quantum entanglement. We know that we do not need these concepts in the vast sub-realm of classical and relativistic physics, but they become relevant outside it.

A question often raised is the *intelligibility* of the *Unus Mundus*. We believe that this question belongs to pre-Galilean metaphysics. Quantum mechanics has reminded us that our only way of knowing is through perception mediated by our senses. Perceived and perceiver are inevitably connected. Ultimately all empirical knowledge of the world is via the conjunction of our faculties to perceive the world and the events that take place either in a laboratory or in our ordinary life. We definitely perceive several aspects of the *Unus Mundus* and therefore it *is* intelligible in the Galilean-Einstein-Popper sense. We can formulate theories about it and use *reasoned experience* to disprove them. This is, after all, the only form of knowledge we are positively sure of. The language of mathematics seems still quite appropriate for this endeavour, and in this sense, if we avoid the pitfall of identifying the model and the modelled, Galileo's conjecture seems to stand. The unintelligibility postulate becomes necessary only if we consider the intrinsic ontic essence of *Unus Mundus* because nothing can be said or known about an all-encompassing entity, in a similar way in which contemplation of the true essence of god leads to negative or apophatic theology.

The fact that Quantum Mechanics is the first physics discipline that has required to go beyond the assumptions of classical and relativistic physics has sparked a considerable interest in attempting to use its concepts and formalism to describe the *Unus Mundus*. It is probably early days for this, in spite of all the egregious minds that have devoted their attention to this problem. It is quite likely that a Galilean-Einstein-Popper theory of the *Unus Mundus* will include Quantum Mechanics as a special case, in the same way in which classical and relativistic physics are a special case of Quantum Mechanics and classical mechanics of relativity. In this sense, it is very interesting to consider recent works by Atmanspacher (2016) trying to define a *weaker* Quantum Mechanics that could be a better basis to describe the Psyche.

The intuition behind this attempt is that *less stringent* conditions may lead to a *more general* theory. This effort is interesting as any reflection on the assumptions of our model of the world is per se useful.

However, if we look at the history of physics, real breakthroughs come, again in Einstein's words, from "the discovery of a universal formal principle". The *generalisation* of classical mechanics came from the introduction of one more absolute (the speed of light). In this sense, the name relativity has been an epistemological misnomer, as this theory introduces one more absolute with respect to Galilean relativity. The *generalisation* of classical physics to the quantum world came also at the *cost* of the introduction of more formal principles such as the non-commutativity of associate operators, Heisenberg's uncertainty and Pauli's exclusion principles. In a similar way, we may expect that only the discovery of new universal principles could lead to a theory encompassing more aspects of our reality, analogously to what has happened for relativity with the introduction of the universality of the speed of light, general relativity with the extension of the equivalence principle and quantum mechanics with the non-commutativity of operators and the quantization of physical quantities.

We might expect that, with each new discovery of a universal principle, a larger class of phenomena be explained and complementary, and possibly incompatible, descriptions of reality, such as general relativity and quantum mechanics today, can be unified. This may be seen as an *epistemic series* that itself, if proven convergent, would constitute an intrinsic ontic definition of the *Unus Mundus*.

This paper does not claim to introduce any new universal principle, but we felt important to share the above ideas to put our work in a wider context. In this work, we will rather follow-up from our previous publication in exploring Quantum Information as a model for the relations between physical and psychical world.

Quantum Information Theory and *Unus Mundus*

In the light of the preceding discussion, we believe that a *formal universal principle* that has not yet yield all its potential is the *Pauli-Jung conjecture*, that is the essential ontic unity of mind and matter, in spite of the obvious and undeniable epistemic split between the two. For a fascinating discussion on this matter see Atmanspacher (2016).

Since the seminal discussions between Pauli and Jung (1955) (see also Meier 2000) there have been several attempts at a formalisation of a possible physical description of the Psyche based on the principles of Quantum Mechanics. A review would be outside the scope of this paper; however, a very good overview can be found in Atmanspacher and Primas (2009).

Much attention has been devoted in recent years to the concept of qubit. A qubit is the simplest possible quantum system, whose states are described by a two-dimensional Hilbert space. A qubit carries a single bit of information according to its orientation with respect to a privileged direction. qubits are the conceptual building block of quantum computing and in the physical world they describe a 1/2 spin system.

Following the same line, in this paper we introduce a very simple model of the interaction between Physis and Psyche. As we have done in previous publications (Galli Carminati and Martin (2008), Martin et al. (2010), (2013)), we will consider an abstract space of quantum information vectors (qubits) each one carrying a single bit of information. The representation of this pure state space of two level quantum systems is often graphically provided via the so-called Bloch sphere (Figure 1).

Although any reasonable macroscopic physics system will need a large number of bits of information to be described, a two-qubit system and its property is very interesting since any many-qubit quantum logic circuit can be constructed out of single-bit and two-bit operations (DiVincenzo (1995), Lloid (1995), Deutsch et al. (1995), Barenco et al. (1995)).

As elements of an abstract space of information we assume that the Physis and the Psyche can be represented by quantum states (Baaquie and Martin (2005), Galli Carminati and Martin (2008), Martin and Galli Carminati (2009)). These quantum states, respectively $|UF\rangle$ and $UP\rangle$, are vectors of Hilbert spaces HF and HP that represent the states of the respective qubits. As the simplest case of a system with a Physis and a Psyche, we introduce the tensor product of the two spaces $HF \otimes HP$, each one containing a single qubit. It is important at this point to note that Hans Primas has studied the Unus Mundus (Primas (2009)). Unlike us who consider tensor product of Hilbert spaces, he considers tensor products of algebras of operators acting on Hilbert spaces

$$\begin{aligned}\Psi &= \alpha\,|0\rangle^{(F)}|0\rangle^{(P)} + \beta\,|0\rangle^{(F)}|1\rangle^{(P)} \\ &+\gamma\,|1\rangle^{(F)}|0\rangle^{(P)} + \delta\,|1\rangle^{(F)}|1\rangle^{(P)}\end{aligned} \tag{1}$$

We note that this system is separable *iff* $\beta/\alpha = \delta/\gamma = k$, because in this case we have:

$$\begin{aligned}\Psi &= \alpha\,|0\rangle^{(F)}|0\rangle^{(P)} + \beta\,|0\rangle^{(F)}|1\rangle^{(P)} + \gamma\,|1\rangle^{(F)}|0\rangle^{(P)} + \delta\,|1\rangle^{(F)}|1\rangle^{(P)} \\ &= \alpha\,|0\rangle^{(F)}|0\rangle^{(P)} + k\alpha\,|0\rangle^{(F)}|1\rangle^{(P)} + \gamma\,|1\rangle^{(F)}|0\rangle^{(P)} + k\gamma\,|1\rangle^{(F)}|1\rangle^{(P)} \\ &= \alpha\,|0\rangle^{(F)}\left(|0\rangle^{(P)} + k\,|1\rangle^{(P)}\right) + \gamma\,|1\rangle^{(F)}\left(|0\rangle^{(P)} + k\,|1\rangle^{(P)}\right) \\ &= \left(\alpha\,|0\rangle^{(F)} + \gamma\,|1\rangle^{(F)}\right)\left(|0\rangle^{(P)} + k\,|1\rangle^{(P)}\right) \\ &= \left(a\,|0\rangle^{(F)} + b\,|1\rangle^{(F)}\right)\left(c\,|0\rangle^{(P)} + d\,|1\rangle^{(P)}\right)\end{aligned} \tag{2}$$

where $\alpha = ac, \beta = ad, \gamma = bc \text{ and } \delta = bd$. It is trivial to verify that if the initial vectors were normalised (i.e. $a^2 + b^2 = c^2 + d^2 = 1$), also Ψ is normalised. We now consider the general Hamiltonian governing the evolution of the system of the two qubits:

$$\mathcal{H} = \begin{pmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \\ h_{3,1} & h_{3,2} & h_{3,3} & h_{3,4} \\ h_{4,1} & h_{4,2} & h_{4,3} & h_{4,4} \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h^*_{1,2} & h_{2,2} & h_{2,3} & h_{2,4} \\ h^*_{1,3} & h^*_{2,3} & h_{3,3} & h_{3,4} \\ h^*_{1,4} & h^*_{2,4} & h^*_{3,4} & h_{4,4} \end{pmatrix} \tag{3}$$

where the elements $h_{i,i}$ are real given the well-known hermiticy properties of the Hamiltonian operator. This operator has 16 degrees of freedom. In case of separable states we can consider two special forms of Hamiltonian, that only affect one of the two qubits and not the other. We define $\mathcal{H}^{(F)}$ as the *physic* Hamiltonian:

$$H^{(F)} = \mathcal{H}^{(F)} \otimes \mathcal{I}^{(P)} = \begin{pmatrix} h_{1,1}^{(F)} & h_{1,2}^{(F)} \\ h_{1,2}^{(F)*} & h_{2,2}^{(F)} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} h_{1,1}^{(F)} & 0 & h_{1,2}^{(F)} & 0 \\ 0 & h_{1,1}^{(F)} & 0 & h_{1,2}^{(F)} \\ h_{1,2}^{(F)*} & 0 & h_{2,2}^{(F)} & 0 \\ 0 & h_{1,2}^{(F)*} & 0 & h_{2,2}^{(F)} \end{pmatrix}$$

$$= \begin{pmatrix} \rho_{1,1}^{(F)} & 0 & \rho_{1,2}^{(F)} e^{i\phi_{1,2}^{(F)}} & 0 \\ 0 & \rho_{1,1}^{(F)} & 0 & \rho_{1,2}^{(F)} e^{i\phi_{1,2}^{(F)}} \\ \rho_{1,2}^{(F)} e^{-i\phi_{1,2}^{(F)}} & 0 & \rho_{2,2}^{(F)} & 0 \\ 0 & \rho_{1,2}^{(F)} e^{-i\phi_{1,2}^{(F)}} & 0 & \rho_{2,2}^{(F)} \end{pmatrix} \quad (4)$$

This Hamiltonian governs the evolution of the physical world in our model and does not affect the Psyche. It has the following (degenerated) eigenvalues:

$$e_1 = \frac{\rho_{2,2}^{(F)} + \rho_{1,1}^{(F)} - \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}}{2}, e_2 = \frac{\rho_{2,2}^{(F)} + \rho_{1,1}^{(F)} + \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}}{2} \quad (5)$$

and eigenvectors

$$0.92! \left[\begin{array}{l} \beta, \alpha, \beta e^{-i\phi_{1,2}^{(F)}} \rho_{2,2}^{(F)} - \rho_{1,1}^{(F)} - \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}\, 2\rho_{1,2}^{(F)}, \alpha e^{-i\phi_{1,2}^{(F)}} \rho_{2,2}^{(F)} - \rho_{1,1}^{(F)} - \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}\, 2\rho_{1,2}^{(F)} \\ \delta, \gamma, \delta e^{-i\phi_{1,2}^{(F)}} \rho_{2,2}^{(F)} - \rho_{1,1}^{(F)} + \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}\, 2\rho_{1,2}^{(F)}, \gamma e^{-i\phi_{1,2}^{(F)}} \rho_{2,2}^{(F)} - \rho_{1,1}^{(F)} + \sqrt{\left(\rho_{1,1}^{(F)} - \rho_{2,2}^{(F)}\right)^2 + 4\rho_{1,2}^{(F)2}}\, 2\rho_{1,2}^{(F)} \end{array} \right] \quad (6)$$

This operator has four degrees of freedom, which leaves us with twelve for the other part. In the same manner, we can consider the *psychic* Hamiltonian $\mathcal{H}^{(P)}$:

$$\mathcal{H}^{(P)} = \mathcal{I}^{(F)} \otimes \mathcal{H}^{(P)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} h_{1,1}^{(P)} & h_{1,2}^{(P)} \\ h_{1,2}^{(P)*} & h_{2,2}^{(P)} \end{pmatrix} = \begin{pmatrix} h_{1,1}^{(P)} & h_{1,2}^{(P)} & 0 & 0 \\ h_{1,2}^{(P)*} & h_{2,2}^{(P)} & 0 & 0 \\ 0 & 0 & h_{1,1}^{(P)} & h_{1,2}^{(P)} \\ 0 & 0 & h_{1,2}^{(P)*} & h_{2,2}^{(P)} \end{pmatrix}$$

$$= \begin{pmatrix} \rho_{1,1}^{(P)} & \rho_{1,2}^{(P)} e^{i\phi_{1,2}^{(P)}} & 0 & 0 \\ \rho_{1,2}^{(P)} e^{-i\phi_{1,2}^{(P)}} & \rho_{2,2}^{(P)} & 0 & 0 \\ 0 & 0 & \rho_{1,1}^{(P)} & \rho_{1,2}^{(P)} e^{i\phi_{1,2}^{(P)}} \\ 0 & 0 & \rho_{1,2}^{(P)} e^{-i\phi_{1,2}^{(P)}} & \rho_{2,2}^{(P)} \end{pmatrix} \quad (7)$$

with the (degenerated) eigenvalues:

$$e_1 = \frac{\rho_{1,1}^{(P)} + \rho_{2,2}^{(P)} - \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2}, e_2 = \frac{\rho_{1,1}^{(P)} + \rho_{2,2}^{(P)} + \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2} \quad (8)$$

and the eigenvectors

$$\begin{bmatrix} \beta, \beta e^{-i\phi_{1,2}^{(P)}} \dfrac{\rho_{2,2}^{(P)} - \rho_{1,1}^{(P)} - \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2\rho_{1,2}^{(P)}}, -\alpha e^{i\phi_{1,2}^{(P)}} \dfrac{\rho_{2,2}^{(P)} - \rho_{1,1}^{(P)} + \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2\rho_{1,2}^{(P)}}, \alpha \\ \delta, \delta e^{-i\phi_{1,2}^{(P)}} \dfrac{\rho_{2,2}^{(P)} - \rho_{1,1}^{(P)} + \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2\rho_{1,2}^{(P)}}, -\gamma e^{i\phi_{1,2}^{(P)}} \dfrac{\rho_{2,2}^{(P)} - \rho_{1,1}^{(P)} - \sqrt{\left(\rho_{1,1}^{(P)} - \rho_{2,2}^{(P)}\right)^2 + 4\rho_{1,2}^{(P)2}}}{2\rho_{1,2}^{(P)}}, \gamma \end{bmatrix} \quad (9)$$

Any pair of linear combinations of, respectively (α, β) and (γ, δ) provides a couple of eigenvectors for the respective eigenvalues. Whatever the choice, the eigenvectors relative to a different eigenvalue are orthogonal. This second operator has also four degrees of freedom, and therefore the remaining part has eight.

Interaction Physis - Psyche

Now let's suppose we have a generic Hamiltonian, describing the evolution in time of psychic and physic reality. We can think this Hamiltonian as composed by three components:

$$\mathcal{H} = \mathcal{H}^{(F)} + \mathcal{H}^{(P)} + \mathcal{H}_I \quad (10)$$

Where H_I is the *interaction* Hamiltonian describing the interaction evolution of Physis and Psyche. Let's start from $H^{(P)}$ setting

$$\mathcal{H}^{(P)} = \begin{pmatrix} \frac{h_{1,1}+h_{3,3}}{2} & \frac{h_{1,2}+h_{3,4}}{2} & 0 & 0 \\ \frac{h_{1,2}^*+h_{3,4}^*}{2} & \frac{h_{2,2}+h_{4,4}}{2} & 0 & 0 \\ 0 & 0 & \frac{h_{1,1}+h_{3,3}}{2} & \frac{h_{1,2}+h_{3,4}}{2} \\ 0 & 0 & \frac{h_{1,2}^*+h_{3,4}^*}{2} & \frac{h_{2,2}+h_{4,4}}{2} \end{pmatrix} \quad (11)$$

we have

$$\begin{aligned} \mathcal{H} &= \mathcal{H}^{(P)} + \begin{pmatrix} \frac{h_{1,1}-h_{3,3}}{2} & \frac{h_{1,2}-h_{3,4}}{2} & h_{1,3} & h_{1,4} \\ \frac{h_{1,2}^*-h_{3,4}^*}{2} & \frac{h_{2,2}-h_{4,4}}{2} & h_{2,3} & h_{2,4} \\ h_{1,3}^* & h_{2,3}^* & -\frac{h_{1,1}-h_{3,3}}{2} & -\frac{h_{1,2}-h_{3,4}}{2} \\ h_{1,4}^* & h_{2,4}^* & -\frac{h_{1,2}^*-h_{3,4}^*}{2} & -\frac{h_{2,2}-h_{4,4}}{2} \end{pmatrix} \\ &= \mathcal{H}^{(P)} + \begin{pmatrix} z_{1,1} & z_{1,2} & h_{1,3} & h_{1,4} \\ z_{1,2}^* & z_{2,2} & h_{2,3} & h_{2,4} \\ h_{1,3}^* & h_{2,3}^* & -z_{1,1} & -z_{1,2} \\ h_{1,4}^* & h_{2,4}^* & -z_{1,2}^* & -z_{2,2} \end{pmatrix} \end{aligned} \quad (12)$$

We now define $\mathcal{H}^{(F)}$

$$\mathcal{H}^{(F)} = \begin{pmatrix} \frac{z_{1,1}+z_{2,2}}{2} & 0 & \frac{h_{1,3}+h_{2,4}}{2} & 0 \\ 0 & \frac{z_{1,1}+z_{2,2}}{2} & 0 & \frac{h_{1,3}+h_{2,4}}{2} \\ \frac{h^*_{1,3}+h^*_{2,4}}{2} & 0 & -\frac{z_{1,1}+z_{2,2}}{2} & 0 \\ 0 & \frac{h^*_{1,3}+h^*_{2,4}}{2} & 0 & -\frac{z_{1,1}+z_{2,2}}{2} \end{pmatrix} \quad (13)$$

We have

$$\mathcal{H} = \mathcal{H}^{(F)} + \mathcal{H}^{(P)} + \begin{pmatrix} \frac{z_{1,1}-z_{2,2}}{2} & z_{1,2} & \frac{h_{1,3}-h_{2,4}}{2} & h_{1,4} \\ z^*_{1,2} & -\frac{z_{1,1}-z_{2,2}}{2} & h_{2,3} & -\frac{h_{1,3}-h_{2,4}}{2} \\ \frac{h^*_{1,3}-h^*_{2,4}}{2} & h^*_{2,3} & -\frac{z_{1,1}-z_{2,2}}{2} & -z_{1,2} \\ h^*_{1,4} & -\frac{h^*_{1,3}-h^*_{2,4}}{2} & -z^*_{1,2} & \frac{z_{1,1}-z_{2,2}}{2} \end{pmatrix} \quad (14)$$

Which leaves us with a $\mathcal{H}_I$ of the form:

$$\mathcal{H}_I = \begin{pmatrix} h^{(I)}_{1,1} & h^{(I)}_{1,2} & h^{(I)}_{1,3} & h^{(I)}_{1,4} \\ h^{i*}_{1,2} & -h^{(I)}_{1,1} & h^{(I)}_{2,3} & -h^{(I)}_{1,3} \\ h^{i*}_{1,3} & h^{i*}_{2,3} & -h^{(I)}_{1,1} & -h^{(I)}_{1,2} \\ h^{i*}_{1,4} & -h^{i*}_{1,3} & -h^{i*}_{1,2} & h^{(I)}_{1,1} \end{pmatrix} = \begin{pmatrix} A & B & C & D \\ B^* & -A & E & -C \\ C^* & E^* & -A & -B \\ D^* & -C^* & -B^* & A \end{pmatrix} \quad (15)$$

This is the generic form of the interaction matrix. We now turn to the question of the effect of this interaction matrix on a non-entangled state. In particular, we want to see whether we can further subdivide $\mathcal{H}_I$ into a non-entangling operator $\mathcal{H}_{Ine}$ and an entangling one $\mathcal{H}_{Ie}$. To discover what is the general form of (15) that is non-entangling, we start from a non-entangled state and to apply this interaction:

$$\begin{pmatrix} A & B & C & D \\ B^* & -A & E & -C \\ C^* & E^* & -A & -B \\ D^* & -C^* & -B^* & A \end{pmatrix} \begin{pmatrix} \alpha \\ k\alpha \\ \gamma \\ k\gamma \end{pmatrix} = \begin{pmatrix} \alpha(A+Bk)+\gamma(C+Dk) \\ \alpha(B^*-Ak)+\gamma(E-Ck) \\ \alpha(C^*+E^*k)-\gamma(A+Bk) \\ \alpha(D^*-C^*k)-\gamma(B^*-Ak) \end{pmatrix} \quad (16)$$

The condition of non-entanglement of the final state can be expressed by equating the product of the first and fourth components to the one of the second and third. In other words:

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} \textit{ is not entangled iff } \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Leftrightarrow \alpha\delta = \beta\gamma \quad (17)$$

and in our case this gives

$$\left[\alpha(A+Bk)+\gamma(C+Dk)\right]\left[\alpha(D^*-C^*k)-\gamma(B^*-Ak)\right] = \\ = \left[\alpha(B^*-Ak)+\gamma(E-Ck)\right]\left[\alpha(C^*+E^*k)-\gamma(A+Bk)\right] \quad (18)$$

If we now solve this equation, we find that the generic form of the interacting and non-entangling matrix is the following:

$$\mathcal{H}_{Ine} = \begin{pmatrix} \rho_A & \rho_B e^{i\phi_B} & \rho_C e^{i\phi_C} & \frac{\rho_B \rho_C}{\rho_A} e^{i(\phi_B+\phi_C)} \\ \rho_B e^{-i\phi_B} & -\rho_A & \frac{\rho_B \rho_C}{\rho_A} e^{-i(\phi_B-\phi_C)} & -\rho_C e^{i\phi_C} \\ \rho_C e^{-i\phi_C} & \frac{\rho_B \rho_C}{\rho_A} e^{i(\phi_B-\phi_C)} & -\rho_A & -\rho_B e^{i\phi_B} \\ \frac{\rho_B \rho_C}{\rho_A} e^{-i(\phi_B+\phi_C)} & -\rho_C e^{-i\phi_C} & -\rho_B e^{-i\phi_B} & \rho_A \end{pmatrix} \quad (19)$$

If we set

$$U = \begin{pmatrix} \rho_A & \rho_B e^{i\phi_B} \\ \rho_B e^{-i\phi_B} & -\rho_A \end{pmatrix}; \quad (20)$$

we can write

$$\mathcal{H}_{Ine} = \begin{pmatrix} U & \frac{\rho_C}{\rho_A} e^{i\phi_C} U \\ \frac{\rho_C}{\rho_A} e^{-i\phi_C} U & -U \end{pmatrix} \quad (21)$$

This Hamiltonian has the following opposite degenerated eigenvalues:

$$e_1 = -\sqrt{\rho_B{}^2 + \rho_A{}^2}\sqrt{\rho_C{}^2 + \rho_A{}^2}\,\rho_A; e_2 = \sqrt{\rho_B{}^2 + \rho_A{}^2}\sqrt{\rho_C{}^2 + \rho_A{}^2}\,\rho_A \quad (22)$$

if we define

$$x = \frac{\rho_A}{\rho_B} \text{and}\, y = \frac{\rho_A}{\rho_C} \quad (23)$$

these can be rewritten as

$$e_1 = -\frac{\rho_B \rho_C}{\rho_A}\sqrt{1+x^2}\sqrt{1+y^2}; e_2 = \frac{\rho_B \rho_C}{\rho_A}\sqrt{1+x^2}\sqrt{1+y^2} \quad (24)$$

Since we have two degenerated eigenvalues, we have two pairs of eigenvectors that we will be able to combine linearly in two subspaces of dimension two. Of all the combination of the pairs of eigenvectors of the same eigenvalue, we can choose those that are composed by pure (non-entangled) states. In this case, it is interesting to note that the eigenvectors belonging to the same eigenvalue are orthogonal. Also, eigenvector belonging to different eigenvalues are orthogonal. Hence, under condition of being separable states, we define an orthogonal basis:

$$\begin{aligned}
&\left[e^{i\phi_C},\ e^{i(\phi_C-\phi_B)}\left(\sqrt{1+x^2}-x\right),\ -\left(\sqrt{1+y^2}+y\right),\ -e^{-i\phi_B}\left(\sqrt{1+x^2}-x\right)\left(\sqrt{1+y^2}+y\right)\right] \\
&\left[e^{i\phi_C},\ -e^{i(\phi_C-\phi_B)}\left(\sqrt{1+x^2}+x\right),\ \sqrt{1+y^2}-y,\ -e^{-i\phi_B}\left(\sqrt{1+x^2}+x\right)\left(\sqrt{1+y^2}-y\right)\right] \\
&\left[e^{i\phi_C},\ e^{i(\phi_C-\phi_B)}\left(\sqrt{1+x^2}-x\right),\ \sqrt{1+y^2}-y,\ e^{-i\phi_B}\left(\sqrt{1+x^2}-x\right)\left(\sqrt{1+y^2}-y\right)\right] \quad (25) \\
&\left[e^{i\phi_C},\ -e^{i(\phi_C-\phi_B)}\left(\sqrt{1+x^2}+x\right),\ -\left(\sqrt{1+y^2}+y\right),\ e^{-i\phi_B}\left(\sqrt{1+x^2}+x\right)\left(\sqrt{1+y^2}+y\right)\right]
\end{aligned}$$

This leaves us with the generic interacting entangling Hamiltonian $\mathcal{H}_{Ie}$ of the form:

$$\mathcal{H}_{Ie} = \begin{pmatrix} 0 & 0 & 0 & \rho_D' e^{i\phi_D'} \\ 0 & 0 & \rho_E' e^{i\phi_E'} & 0 \\ 0 & \rho_E' e^{-i\phi_E'} & 0 & 0 \\ \rho_D' e^{-i\phi_D'} & 0 & 0 & 0 \end{pmatrix} \quad (26)$$

This Hamiltonian has the following eigenvalues:

$$e_1 = -\rho_E'; e_2 = \rho_E'; e_3 = -\rho_D'; e_4 = \rho_D' \quad (27)$$

It is interesting to note that the corresponding eigenvectors are four orthogonal Bell-like states

$$\begin{aligned} &\frac{1}{\sqrt{2}}(|0\rangle^{(F)}|1\rangle^{(P)} - e^{-i\phi_E'}|1\rangle^{(F)}|0\rangle^{(P)}) \\ &\frac{1}{\sqrt{2}}(|0\rangle^{(F)}|1\rangle^{(P)} + e^{-i\phi_E'}|1\rangle^{(F)}|0\rangle^{(P)}) \\ &\frac{1}{\sqrt{2}}(|0\rangle^{(F)}|0\rangle^{(P)} - e^{-i\phi_D'}|1\rangle^{(F)}|1\rangle^{(P)}) \\ &\frac{1}{\sqrt{2}}(|0\rangle^{(F)}|0\rangle^{(P)} + e^{-i\phi_D'}|1\rangle^{(F)}|1\rangle^{(P)}) \end{aligned} \quad (28)$$

Entropy and entanglement

It is interesting at this point to define a measure of the entanglement and see how it evolves with the Hamiltonians we have defined above. Such a measure can be easily provided by entropy. If we start from a 2-qubits system we have defined the usual density matrix as: 195

$$\rho = \sum_{i=0,1;\, j=0,1} |ij\rangle a_i^* b_j \langle ij|$$

The total entropy of the system is

$$S \equiv -\mathrm{tr}(\rho \log \rho)$$

With this formalism, the entanglement entropy is defined as the Von Neumann entropy of the reduced matrix:

$$S_A \equiv -\mathrm{tr}(\rho_A \log \rho_A) = -\mathrm{tr}(\rho_B \log \rho_B) \equiv S_B$$

If we define a general separable state as:

$\Psi = \cos(\theta_1/2)\cos(\theta_2/2)|00\rangle + e^{i\phi_2}\cos(\theta_1/2)\sin(\theta_2/2)|01\rangle +$
$e^{i\phi_1}\sin(\theta_1/2)\cos(\theta_2/2)|10\rangle + e^{i(\phi_1+\phi_2)}\sin(\theta_1/2)\sin(\theta_2/2)|11\rangle$ the corresponding density matrix is:

$$\begin{pmatrix} \cos^2(\theta_1/2)\cos^2(\theta_2/2) & e^{i\phi_2}\cos^2(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) \\ e^{-i\phi_2}\cos^2(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) & \cos^2(\theta_1/2)\sin^2(\theta_2/2) \\ e^{-i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2)\cos^2(\theta_2/2) & e^{i(\phi_2-\phi_1)}\cos(\theta_1/2)\sin(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) \\ e^{-i(\phi_2+\phi_1)}\cos(\theta_1/2)\sin(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) & e^{-i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2)\sin^2(\theta_2/2) \end{pmatrix}$$

$$\begin{pmatrix} e^{i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2)\cos^2(\theta_2/2) & e^{i(\phi_2+\phi_1)}\cos(\theta_1/2)\sin(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) \\ e^{i(\phi_1-\phi_2)}\cos(\theta_1/2)\sin(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) & e^{i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2)\sin^2(\theta_2/2) \\ \sin^2(\theta_1/2)\cos^2(\theta_2/2) & e^{i\phi_2}\sin^2(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) \\ e^{-i\phi_2}\sin^2(\theta_1/2)\cos(\theta_2/2)\sin(\theta_2/2) & \sin^2(\theta_1/2)\sin^2(\theta_2/2) \end{pmatrix} \quad (29)$$

With some algebra, it is possible to verify that

$$\rho^2 = \rho$$

this is one of the property of the density matrix, which is idempotent. Note that this means that the eigenvalues of this matrix are either 0 or 1. If we represent the matrix as

$$\begin{pmatrix} a^*a & a^*b & a^*c & a^*d \\ b^*a & b^*b & b^*c & b^*d \\ c^*a & c^*b & c^*c & c^*d \\ d^*a & d^*b & d^*c & d^*d \end{pmatrix}$$

the trace on the states of B gives us

$$\rho_A = \begin{pmatrix} a^*a + b^*b & a^*c + b^*d \\ c^*a + d^*b & c^*c + d^*d \end{pmatrix} \qquad (30)$$

which, given ((29)) gives

$$\rho_A = \begin{pmatrix} \cos^2(\theta_1/2) & e^{i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2) \\ e^{-i\phi_1}\cos(\theta_1/2)\sin(\theta_1/2) & sin^2(\theta_1/2) \end{pmatrix}$$

In order to calculate the entanglement entropy of this pure and separable state, we have to calculate:

$$S_A \equiv -\mathrm{tr}(\rho_A \log \rho_A)$$

To calculate the logarithm of a matrix we will calculate the matrix T that diagonalise ρ so that $\rho' = T^\dagger \rho T$ is diagonal and then note that

$$\begin{aligned} \log(\rho) &= TT^\dagger \log(\rho) TT^\dagger \\ &= T \log(\rho') T^\dagger \end{aligned}$$

where ρ' is the diagonal density matrix. In our case the eigenvalues are 1 and 0 and the corresponding eigenvectors are:

$$e_1(1) = \begin{pmatrix} \cos(\theta_1/2) \\ e^{-i\phi_1}\sin(\theta_1/2) \end{pmatrix}; e_2(0) = \begin{pmatrix} \sin(\theta_1/2) \\ -e^{-i\phi_1}\cos(\theta_1/2) \end{pmatrix};$$ that gives us

$$T = \begin{pmatrix} \cos(\theta_1/2) & \sin(\theta_1/2) \\ e^{-i\phi_1}\sin(\theta_1/2) & -e^{-i\phi_1}\cos(\theta_1/2) \end{pmatrix}$$

and calculating S_A we have

$$\begin{aligned} S_A &= T\mathrm{tr}(\rho' \log \rho')T^\dagger \\ &= \begin{pmatrix} \cos(\theta_1/2) & \sin(\theta_1/2) \\ e^{-i\phi_1}\sin(\theta_1/2) & -e^{-i\phi_1}\cos(\theta_1/2) \end{pmatrix} \mathrm{tr}\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \log(1) & 0 \\ 0 & \log(0) \end{pmatrix}\right]\begin{pmatrix} \cos(\theta_1/2) & e^{i\phi_1}\sin(\theta_1/2) \\ \sin(\theta_1/2) & -e^{i\phi_1}\cos(\theta_1/2) \end{pmatrix} \end{aligned}$$

a logarithm of 0 is not calculable, but if we replace 0 with ε for the second diagonal element, and we compute the matrix multiplication, we can then use the fact that $\lim_{\varepsilon \to 0} \varepsilon \log(\varepsilon) \to 0$. The entanglement entropy is therefore 0, as it should for a separable state.

We can now discuss how does the total entropy evolve. We remember that

$$i\hbar \frac{\partial \rho}{\partial t} = [\rho H] \Rightarrow \frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\rho H]$$

So we can write:

$$\frac{\partial S}{\partial t}=\frac{\partial}{\partial t}\mathrm{tr}\rho\log(\rho)=\mathrm{tr}\left[\frac{\partial\rho}{\partial t}(\log(\rho)+1)\right]=-\frac{i}{\hbar}\mathrm{tr}\left[(\rho H-H\rho)(\log(\rho)+1)\right]$$

$$=-\frac{i}{\hbar}\left(\mathrm{tr}\left(\rho H\left(\log(\rho)+1\right)\right)-\mathrm{tr}\left(H\rho\left(\log(\rho)+1\right)\right)\right)$$

now we consider the following

$$\log(\rho)\rho=\left(\sum_{i=0,\infty}a_i\rho^i\right)\rho=\sum_{i=0,\infty}a_i\rho^{i+1}=\rho\left(\sum_{i=0,\infty}a_i\rho^i\right)=\rho\log(\rho)$$ we can write

$$\frac{\partial S}{\partial t}=-\frac{i}{\hbar}\left(\mathrm{tr}\left(\rho H\left(\log(\rho)+1\right)\right)-\mathrm{tr}\left(H\rho\left(\log(\rho)+1\right)\right)\right)$$
$$=-\frac{i}{\hbar}\left(\mathrm{tr}\left(\rho H\left(\log(\rho)+1\right)\right)-\mathrm{tr}\left(H\left(\log(\rho)+1\right)\rho\right)\right)$$
$$=-\frac{i}{\hbar}\left(\mathrm{tr}\left(\rho H\left(\log(\rho)+1\right)\right)-\mathrm{tr}\left(\rho H\left(\log(\rho)+1\right)\right)\right)=0$$

where we have exploited the fact that the trace of a matrix product is invariant under the cyclical permutation of the matrices. This result tells us that the total entropy of an isolated two-qubit system is constant in time.

If now we consider equation ((30)) in the case of a generic state, it is interesting to note how this is not necessarily a true density matrix. In fact, if we calculate ρ_A^2 we obtain:

$$\rho_A^2=\begin{pmatrix}\left(|a|^2+|b|^2\right)^2+|a|^2|c|^2+|b|^2|d|^2+ab^*c^*d+a^*bcd^* & (a^*c+b^*d)\left(|a|^2+|b|^2+|c|^2+|d|^2\right)\\ (c^*a+d^*b)\left(|a|^2+|b|^2+|c|^2+|d|^2\right) & \left(|c|^2+|d|^2\right)^2+|a|^2|c|^2+|b|^2|d|^2+ab^*c^*d+a^*bcd^*\end{pmatrix}$$

if we now recall that $|a|^2+|b|^2+|c|^2+|d|^2=1$, after some algebra we have:

$$\rho_A^2=\begin{pmatrix}|a|^2+|b|^2-|ad-bc|^2 & a^*c+b^*d\\ c^*a+d^*b & |c|^2+|d|^2-|ad-bc|^2\end{pmatrix}=\rho_A-\begin{pmatrix}|ad-bc|^2 & 0\\ 0 & |ad-bc|^2\end{pmatrix}$$

The partial trace is a true density matrix only if the initial states are not entangled, i.e. $ad=cb$. The quantity $|ad-bc|^2$ is in this context a quantity characterising the entanglement of the two states.

Discussion

In our very simple model we have described a combination of two qubits, one supposed to represent the Physis and the other the Psyche. The evolution of this system is governed by a Hamitonian that we have decomposed into four components. The first two components, which are called *local* components, govern the separate evolution of the Physis and the Psyche. The eigenvalues of both these Hamiltonians are degenerated. This is qualitatively understandable since each of these two Hamiltonians operate only on half the Hilbert space.

Further we have identified two "interaction" Hamiltonians: one ($\mathcal{H}_{Ine}$, Formula (19)) that preservers the state of non-entanglement between Physis and Psyche and one ($\mathcal{H}_{Ie}$) that, operating on a non-entangled state, transforms it into one where Physis and Psyche are entangled.

The *non-entangling* Hamiltonian $\mathcal{H}_{Ine}$ has two degenerated eigenvalues of opposite sign and same magnitude: for each eigenvalue it is possible to determine two eigenvectors that represent non-entangled states, orthogonal to each another. Eigenvectors corresponding to different eigenvalues are of course orthogonal. This gives us a orthonormal base whose non-entanglement state is preserved by the Hamiltonian $\mathcal{H}_{Ine}$.

The remaining Hamiltonian, $\mathcal{H}_{Ie}$ (see Formula (26)) has four different eigenvalues, two pairs of the same magnitude and opposite sign, and its eigenvectors are Bell-like states.

In our very simple model, Physis and Psyche can evolve separately with no interaction with each other thanks to their "local" Hamiltonians. In case of a couple of qubits, each has two *frequencies* (one for each of the two degenerated eigenvalues) of evolution. This can be seen as the description of the working of the physical world and to the inner activity of our Psyche when they do not interact with each other. This is consistent with the perceived *dualism* between Physis and Psyche and we can consider that it expresses those laws that only apply to Physis or to Psyche. These frequencies of evolution are linked to Chronos (Χρόνος), the time in which the events happen and that we perceive as passing. We intuitively divide this time in a past and a future separated by an intuition of now-ness that we call present. Physis and Psyche can evolve at different paces, and this corresponds well to our everyday's experience of a "psychical" and "physical" time.

Non-entangled states of Physis and Psyche can evolve and interact via $\mathcal{H}_{Ine}$ maintaining their state of non-entanglement. The eigenvalues of the corresponding Hamiltonian are opposite, i.e. they describe a *symmetric* time evolution. The corresponding eigenvectors are orthogonal. This can be considered to express the common evolution of Physis and Psyche. It is the *cosmic pulse* of the *Unus Mundus*. Time flows in both directions with the same pulse, since the *Unus Mundus* is timeless and eternal, but still full of activity. The eigenvalues are degenerated, and this is suggestive of the world of archetypes. Archetypes are timeless (this is why sometimes it is said that "there is no time in the *Unus Mundus*") and this is expressed by the time symmetry of $\mathcal{H}_{Ine}$. They are double, as couples of eigenvectors corresponding to the same eigenvalue, and they are *orthogonal*. Archetypes are combinations of Psyche and Physis and evolve timelessly without generating synchronicity as pure states. This holistic picture is at odd with the Platonic view which is essentially dualistic, and it reinforces the vision of archetypes as building elements of the *Unus Mundus* in its entirety, with no distinction between Physis and Psyche.

It is tempting to further our discussion here and observe that if this were the only Hamiltonian governing the *Unus Mundus*, there would be a "pulse", but no entanglement, which would negate all possibility of measure and of perception. In some sense, this could be the un-knowledgeable ontic essence of the *Unus Mundus* supposed by Atmanspacher. In this state, time would have no possible connection with a "perceiver". This is the Aïon (Aἰών) time, on which Jung has written an essay (Jung (1969)), the circular and eternal time of the universe considered by the French philosopher Gilles Deleuze as the opposite of Chronos. It is a time of the pure moment which never ceases to divide itself into unlimited past and future: "the whole line of Aïon is traversed by the moment, which never ceases to move on it and it is never at its own place" (Deleuze (1969)). If the Aïon moment "is never at its own place", it is because Aïon is pure becoming, unidentifiable, undetectable, in which time ceases to divide itself into a before and an after, and flows without being able to be measured. This "eternal instant" is the proper time of the *Unus Mundus*. Possibly also space in the *Unus Mundus* "is never at its own place" and also space "flows" unidentifiable and undetectable. In this sense, we can say that space and time do not exist in the *Unus Mundus* in the sense they exist in our world.

This picture alone would not be complete however, and for two reasons. First of all, in spite of their dual nature, archetypes are not indistinguishable, so there must be an interaction that separates them, in a similar way in which electron spin states are separate by the spin interactions to reveal the fine structure. This is the role of $\mathcal{H}_{Ie}$ (Formula (26)). Here all degeneracy is removed and Physis and Psyche are entangled. The eigenvectors of this Hamiltonian are maximally entangled Bell-like states. This is the source of entanglement and synchronicity and it introduces what the Greek called the "supreme moment", the time of Kairos (καιρός) in the combined evolution of Physis and Psyche. Kairos is the time of the acausal correlation identified by Jung, when the archetypes "emerge" into the perceptible world. It is the time of the "opportunity" that transcends and combines

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Chronos and Aïon. Thanks to $\mathcal{H}_{Ie}$ synchronicity is introduced in the Unus Mundus, as pairs of physical and psychical events significantly correlated. Every time a $\mathcal{H}_{Ie}$ interaction happens, an entangled state is created which when "measured" (i.e. perceived) by the subject creates a significant correspondence between a Psychic and a Physic state, in the sense that the "measure" of one of the two affects the state of the other. This is the essence of Jung's scarab.

The *amount* of synchronicity in the world depends on the amount of entanglement that was there initially and on the magnitude of the $\mathcal{H}_{Ie}$ component in the total $\mathcal{H}$ at a given moment, which regenerates synchronicity. This in turn depends on the initial state in which the Universe has been *prepared* by the Big Bang and on the possible dependence of $\mathcal{H}$ on time (and space?). This is also function of the relative *magnitude* of the components of $\mathcal{H}_{Ine}$ and $\mathcal{H}_{Ie}$. The question here is of philosophical nature. If synchronicty is a rare event, then we can postulate that the magnitude of $\mathcal{H}_{Ie}$ is small as compared to $\mathcal{H}_{Ine}$ and non-entangling, non-synchronous interactions act as a *background* against which synchronicity is perceived as special and stands out. The initial entanglement created by the Big Bang is then progressively washed away by decoherence and observation and little new synchronicity is created.

If on the contrary, the relative magnitude of $\mathcal{H}_{Ie}$ is large, the synchronicity is continuously recreated and is ubiquitous, being present everywhere all the time. In this case only our perception of synchronicity would be limited.

But there is another fundamental question that we should consider in this context. Is the Psychic component of the *Unus Mundus* an invariant, present since the Big Bang, or it is growing with the appearance and development of consciousness?

Conclusions

We have developed the simplest possible model of the interaction Physis Psyche as a product of two Hilbert spaces each one with a single qubit. The analysis of the Hamiltonian governing the evolution of this system has lead us to some suggestive results on the relations between Physis and Psyche. The model seems to offer a simple description of archetypes as degenerated states and all eigenvalues come in opposite pairs, as if time could flow in both directions. All this is very suggestive of the fundamental nature of the *Unus Mundus* as described by G. Dorn centuries ago. It is doubtful whether such a model could provide more details or if more quantitative conclusions could be drawn, but we believe that the results reached bring a contribution to this interesting subject.

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